Logistic regression II

Slides: initially developed by Mine Çetinkaya-Rundel and Curry W. Hilton of OpenIntro

The logistic regression model

In the logistic regression model: logit(p) = log(p / (1-p)) is **linear** in x_i's.

$$y_i \sim \text{Binom}(p_i)$$

$$\eta = \beta_0 + \beta_1 x_1 + \cdots + \beta_n x_n$$

$$\text{logit}(p) = \eta$$

Binom(p_i): "probability of success = p_i". From which we arrive at,

$$p_i = \frac{\exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_n x_{n,i})}{1 + \exp(\beta_0 + \beta_1 x_{1,i} + \cdots + \beta_n x_{n,i})}$$

Donner party data

Let’s revisit the Donner party data from Monday

Recall we have variables:

- Survived (categorical)
- Age
- Sex
- Name

Let’s formulate a logistic regression model for predicting survival using Age and Sex. And then let’s examine a hypothesis test for whether or not Age is significant for predicting the log-odds of survival.

Hypothesis test for the whole model

```
summary(glm(Status ~ Age + Sex, data=donner, family=binomial))

## Call:
## glm(formula = Status ~ Age + Sex, family = binomial, data = donner)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  1.63312    1.11018   1.471   0.1413
## Age         -0.07820    0.03728  -2.097   0.0359 *
## SexFemale    1.59729    0.75547   2.114   0.0345 *
## ---
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 61.827  on 44  degrees of freedom
## Residual deviance: 51.256  on 42  degrees of freedom
## AIC: 57.256
##
## Number of Fisher Scoring iterations: 4
```

You can ignore the stuff below ---, beyond scope of course.

Hypothesis tests for a coefficient

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	1.6331	1.1102	1.47	0.1413
Age	-0.0782	0.0373	-2.10	0.0359
SexFemale	1.5973	0.7555	2.11	0.0345

We are still able to perform inference on individual coefficients, the basic setup is exactly the same as what we've seen before except we use a Z-test.

Note: Beyond the scope of this course to describe how standard error is calculated.

Testing for the slope of Age

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	1.6331	1.1102	1.47	0.1413
Age	-0.0782	0.0373	-2.10	0.0359
SexFemale	1.5973	0.7555	2.11	0.0345

$$H_0 : \beta_{age} = 0$$

$$H_A : \beta_{age} \neq 0$$

$$Z = \frac{\hat{\beta_{age}} - \beta_{age}}{SE_{age}} = \frac{-0.0782 - 0}{0.0373} = -2.10$$

$$\text{p-value} = P(|Z| > 2.10) = P(Z > 2.10) + P(Z < -2.10)$$

$$= 2 \times 0.0178 = 0.0359$$

Confidence interval for age slope coefficient

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	1.6331	1.1102	1.47	0.1413
Age	-0.0782	0.0373	-2.10	0.0359
SexFemale	1.5973	0.7555	2.11	0.0345

Remember, the interpretation for a slope is the change in log odds ratio per unit change in the predictor.

We can create confidence interval using point est. +/- margin of error.

Log odds ratio CI if we want 95% confidence (z* = 1.96):

$$CI = PE \pm CV \times SE = -0.0782 \pm 1.96 \times 0.0373 = (-0.1513, -0.0051)$$

Odds ratio:

$$\exp(CI) = (\exp(-0.1513), \exp(-0.0051)) = (0.8596, 0.9949)$$

Example - Birdkeeping and Lung Cancer

A 1972 - 1981 health survey in The Hague, Netherlands, discovered an association between keeping pet birds and increased risk of lung cancer. To investigate birdkeeping as a risk factor, researchers conducted a case-control study of patients in 1985 at four hospitals in The Hague (population 450,000). They identified 49 cases of lung cancer among the patients who were registered with a general practice, who were age 65 or younger and who had resided in the city since 1965. They also selected 98 controls from a population of residents having the same general age structure.

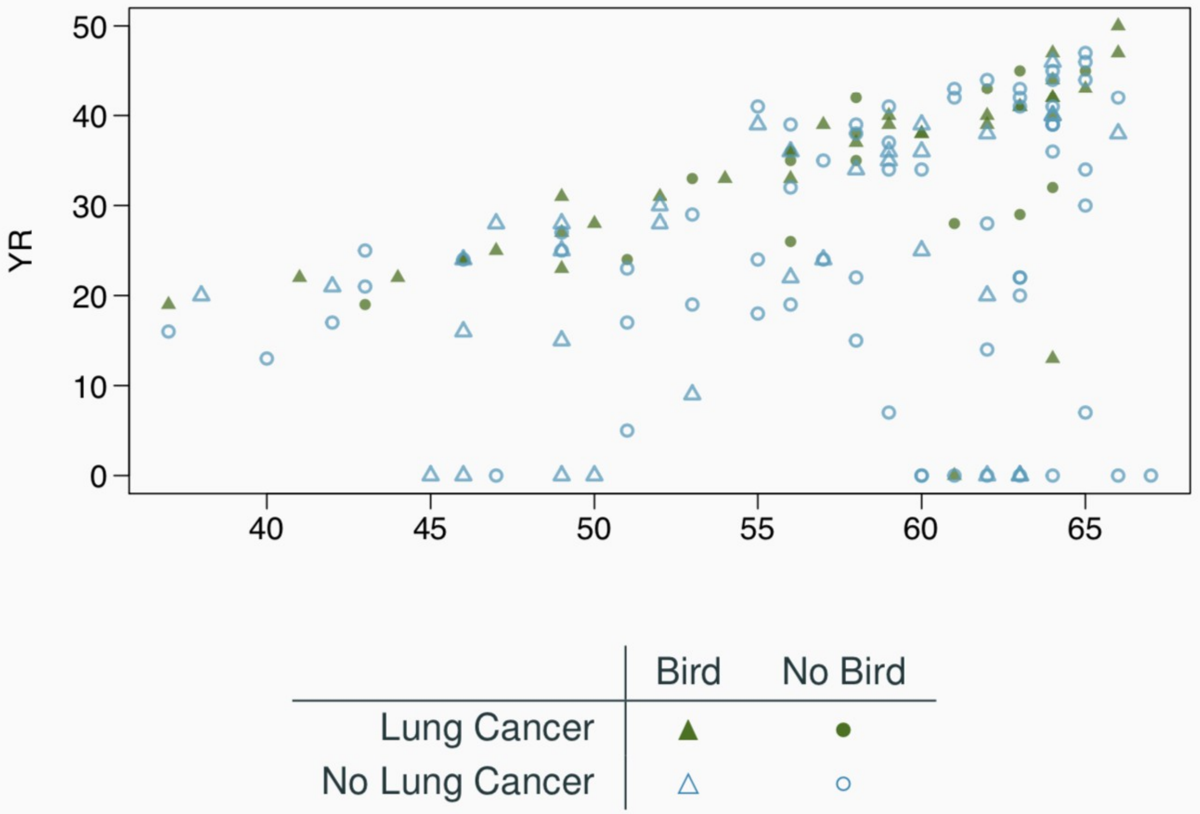
From *Ramsey, F.L. and Schafer, D.W. (2002). The Statistical Sleuth: A Course in Methods of Data Analysis (2nd ed)*

Example - Birdkeeping and Lung Cancer - Data

	LC	FM	SS	BK	AG	YR	CD
1	LungCancer	Male	Low	Bird	37.00	19.00	12.00
2	LungCancer	Male	Low	Bird	41.00	22.00	15.00
3	LungCancer	Male	High	NoBird	43.00	19.00	15.00
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
147	NoCancer	Female	Low	NoBird	65.00	7.00	2.00

LC	Whether subject has lung cancer
FM	Sex of subject
SS	Socioeconomic status
BK	Indicator for birdkeeping
AG	Age of subject (years)
YR	Years of smoking prior to diagnosis or examination
CD	Average rate of smoking (cigarettes per day)

Example - Birdkeeping and Lung Cancer - EDA



Example - Birdkeeping and Lung Cancer - Model

```
summary(glm(LC ~ FM + SS + BK + AG + YR + CD, data=bird, family=binomial))
## Call:
## glm(formula = LC ~ FM + SS + BK + AG + YR + CD, family = binomial,
##     data = bird)
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.93736  1.80425    -1.074 0.282924
## FMFemale     0.56127  0.53116     1.057 0.290653
## SSHigh       0.10545  0.46885     0.225 0.822050
## BKBird       1.36259  0.41128     3.313 0.000923 ***
## AG          -0.03976  0.03548    -1.120 0.262503
## YR           0.07287  0.02649     2.751 0.005940 **
## CD           0.02602  0.02552     1.019 0.308055
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 187.14  on 146  degrees of freedom
## Residual deviance: 154.20  on 140  degrees of freedom
## AIC: 168.2
```

Example - Birdkeeping and Lung Cancer - Interpretation

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	-1.9374	1.8043	-1.07	0.2829
FMFemale	0.5613	0.5312	1.06	0.2907
SSHigh	0.1054	0.4688	0.22	0.8221
BKBird	1.3626	0.4113	3.31	0.0009
AG	-0.0398	0.0355	-1.12	0.2625
YR	0.0729	0.0265	2.75	0.0059
CD	0.0260	0.0255	1.02	0.3081

Keeping all other predictors constant then,

- The odds ratio of getting lung cancer for bird keepers vs non-bird keepers is exp(1.3626) = 3.91.
- The odds ratio of getting lung cancer for an additional year of smoking is exp(0.0729) = 1.08.

What do the numbers not mean ...

The most common mistake made when interpreting logistic regression is to treat an odds ratio as a ratio of probabilities.

Bird keepers are <u>not</u> 4x more likely to develop lung cancer than non-bird keepers.

This is the difference between **relative risk (RR)** and **an odds ratio (OR)**.

$$RR = \frac{P(\text{disease}|\text{exposed})}{P(\text{disease}|\text{unexposed})}$$

$$OR = \frac{P(\text{disease}|\text{exposed})/[1 - P(\text{disease}|\text{exposed})]}{P(\text{disease}|\text{unexposed})/[1 - P(\text{disease}|\text{unexposed})]}$$

Back to the birds

What is probability of lung cancer in a bird keeper if we knew that P(lung cancer|no birds) = 0.05?

$$OR = \frac{P(\text{lung cancer|birds})/[1 - P(\text{lung cancer|birds})]}{P(\text{lung cancer|no birds})/[1 - P(\text{lung cancer|no birds})]}$$

$$= \frac{P(\text{lung cancer|birds})/[1 - P(\text{lung cancer|birds})]}{0.05/[1 - 0.05]} = 3.91$$

$$P(\text{lung cancer|birds}) = \frac{3.91 \times \frac{0.05}{0.95}}{1 + 3.91 \times \frac{0.05}{0.95}} = 0.171$$

$$RR = P(\text{lung cancer|birds})/P(\text{lung cancer|no birds}) = 0.171/0.05 = 3.41$$

Practice:

9.8. **Spam filtering, prediction.** Recall running a logistic regression to aid in spam classification for individual emails. In this exercise, we've taken a small set of the variables and fit a logistic model with the following output:

term	estimate	std.error	statistic	p.value
`(Intercept)`	-0.81	0.09	-9.34	<0.0001
`to_multiple1`	-2.64	0.30	-8.68	<0.0001
`winneryes`	1.63	0.32	5.11	<0.0001
`format1`	-1.59	0.12	-13.28	<0.0001
`re_subj1`	-3.05	0.36	-8.40	<0.0001

a. Write down the model using the coefficients from the model fit.

b. Suppose we have an observation where `to_multiple` $= 0$, `winner` $= 1$, `format` $= 0$, and `re_subj` $= 0$. What is the predicted probability that this message is spam?

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a. Write down the model using the coefficients from the model fit.

b. Suppose we have an observation where `to_multiple` $= 0$, `winner` $= 1$, `format` $= 0$, and `re_subj` $= 0$. What is the predicted probability that this message is spam?

c. Put yourself in the shoes of a data scientist working on a spam filter. For a given message, how high must the probability a message is spam be before you think it would be reasonable to put it in a *spambox* (which the user is unlikely to check)? What tradeoffs might you consider? Any ideas about how you might make your spam-filtering system even better from the perspective of someone using your email service?

Suppose we had data for attending grad school as a function of GPA and number of year to graduate, and we formed a logistic regression model, which had the following (incomplete!) table.

Term	Estimate	Std Error	Z value	P(>|z|)
(intercept)	0.52	0.02		
GPA	3.0	0.7		
YearsToGrad	-1.5	0.5		

Can you fill out the rest of the table? Do you need additional information/tools? Describe how to fill as much as you can, and describe any issues you run into.

Final exam topics

* Mutate, group by, summarize, date / time
* regex, including str_remove, str_replace, str_replace_all
* pivot_longer, pivot_wider
* joining (left_join, anti_join, semi_join)
* plotting: ggplot, geom_{bar,point,histogram,boxplot}, color / shape / groupings
* functions – incl. using {{embrace}}, default values, `across()`, `if_any()`, `if_all()`
* linear regression - residuals, correlation, least squares line, `lm()`, R^2 and concepts of SST/SSE, categorical variables in linear regression, adjusted R^2 and model selection
* hypothesis tests / confidence intervals / p-values using randomization tests, bootstrap sampling, bootstrap confidence intervals, mathematical approaches (normal distribution / t distribution / F distribution)
  - estimating a single proportion
  - comparing two proportions
  - estimating a single mean
  - comparing two means
  - comparing multiple means
  - slope in linear regression
  - logistic regression